COMMISSION TO STUDY SCHOOL FUNDING| OCTOBER 2020

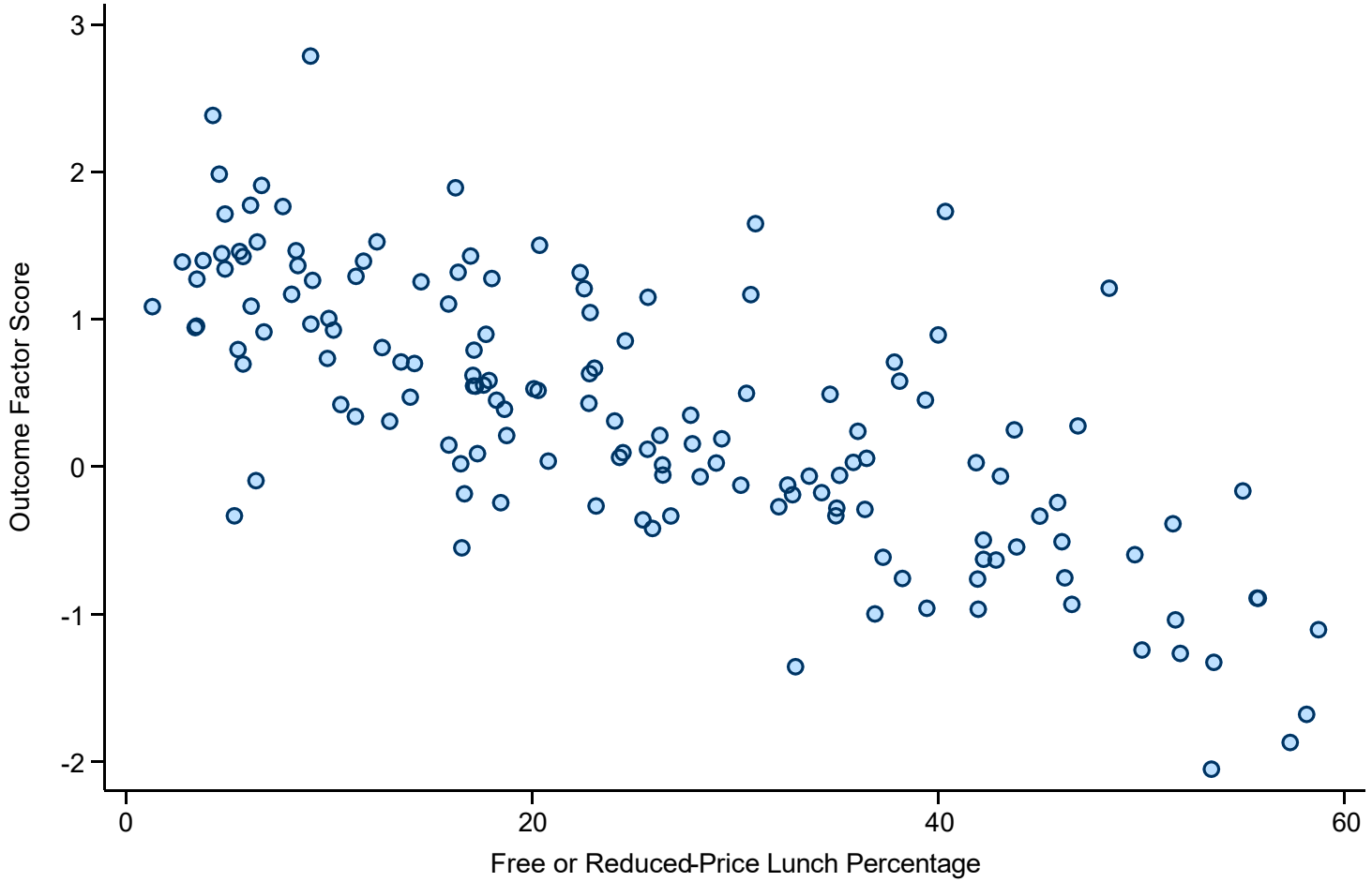
REGRESSION AND COST MODEL RESULTS

Drew Atchison | Jesse Levin

MAKING RESEARCH RELEVANT

Explanation of Simple Regression

Relationship Between Student Outcomes and FRPL Percentage



Regression can be used to quantify the relationship between student outcomes and FRPL.

It does this by fitting a line through the points that best describes the data as follows:

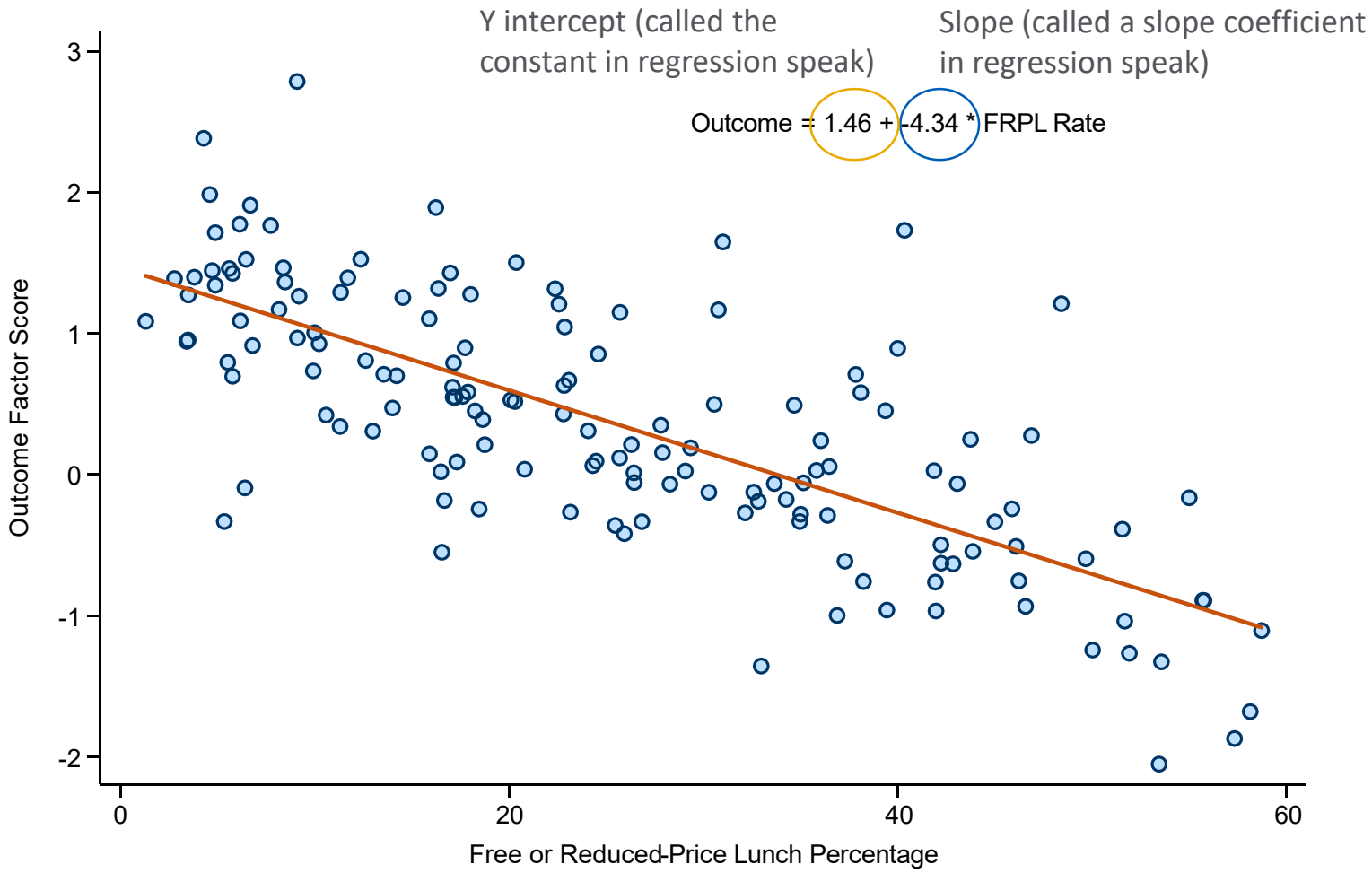
$Y = m*X + b$

Simple Regression Model:

Outcome = $B_0 + B_1$*FRPL Rate + Error

Y = Outcome
m = B_1 (slope coefficient)
x = FRPL Rate
b = B_0 (constant or Y intercept)

Relationship Between Student Outcomes and FRPL Percentage



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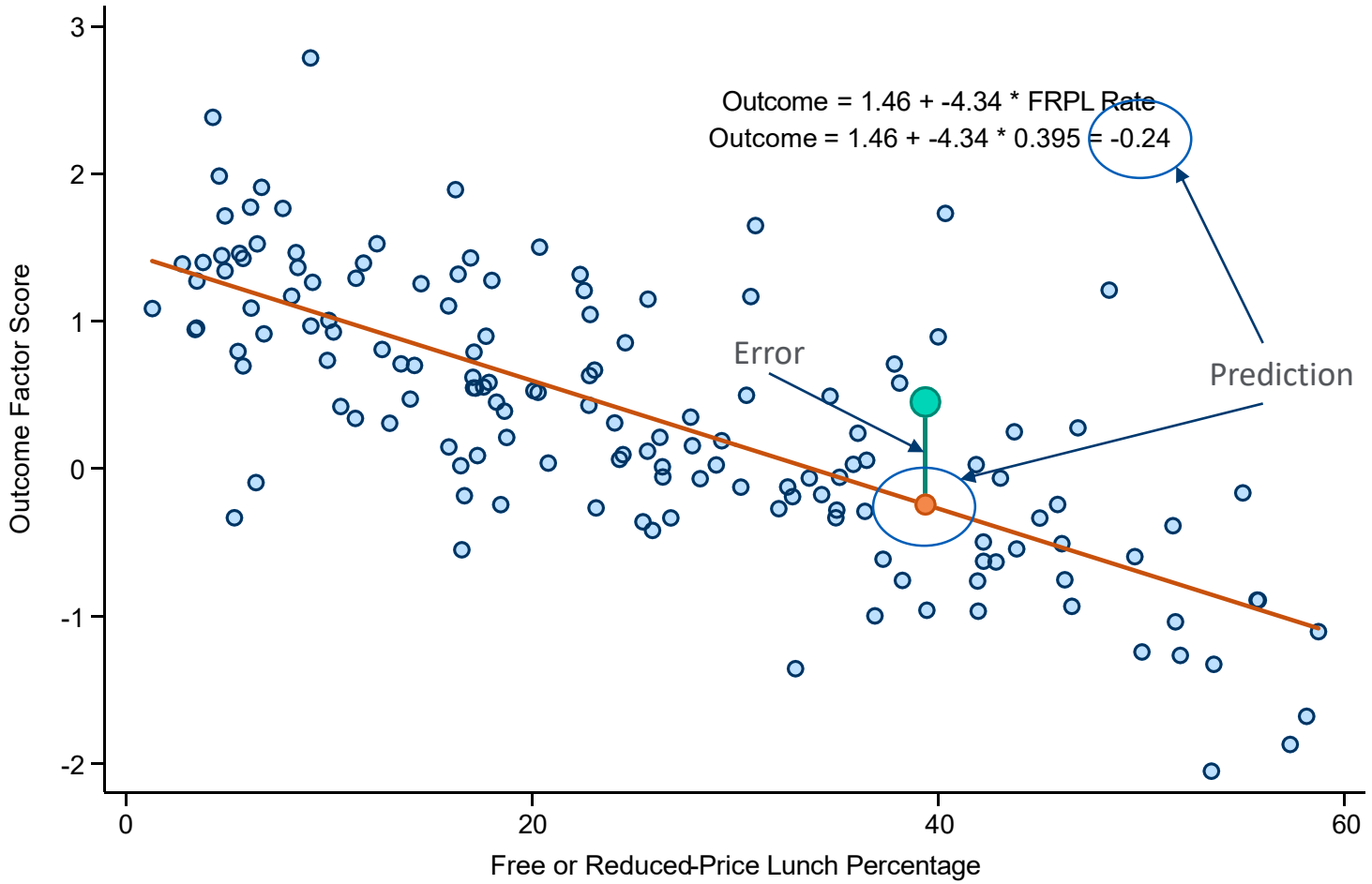
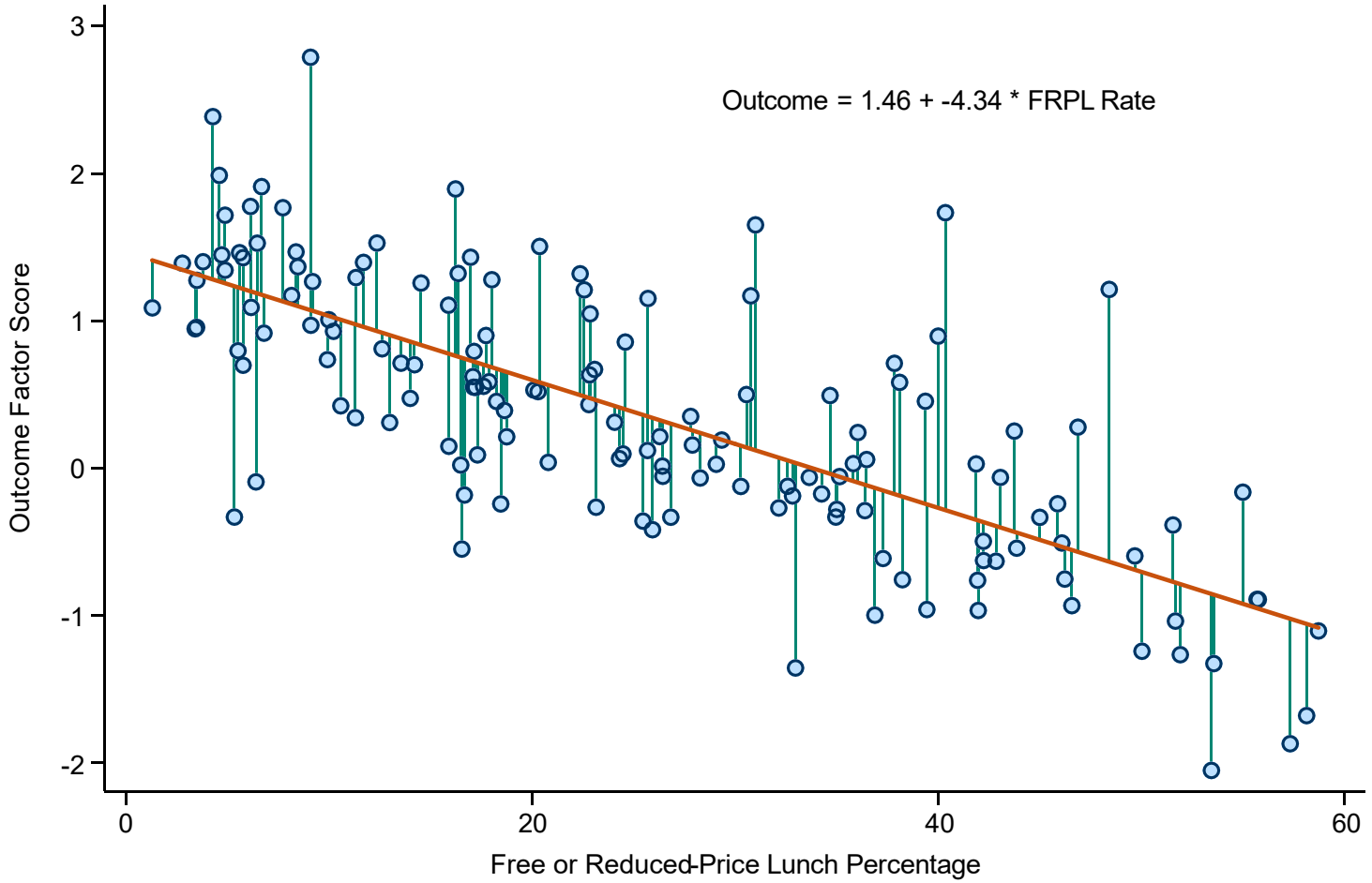


Once the model is estimated, the intercept and slope coefficients can be used to generate a prediction for each observation.

Example: A district with an FRPL rate of 39.5% has an expected outcome that is -0.24.

The difference between the prediction and the observed outcome is the "error" or "residual."

Relationship Between Student Outcomes and FRPL Percentage



The regression model works by estimating the line that minimizes the sum of the errors across all observations.

This example uses only a single variable for simplicity and because it can easily be illustrated.

However, the same principals apply for multivariate regression. The regression model estimates the combination of relationships that generate the best predictions (those with the lowest error).

Cost Model Results

Cost Model

Conceptual Model

Spending Per Pupil = f(Outcome, Student Needs, Enrollment Size, Grade Shares, Staffing Price Level, Efficiency Controls, Year)

Mathematical Model

$$\ln(\text{Spending Per Pupil}) = B_0 + B_1*\text{Outcome} + B_2*\text{FRPL} + B_3*\text{Special Education} + B_4*\text{EL} + B_5*\text{Enroll<=200} + B_6*\text{Enroll201-600} + B_7*\text{Enroll601-1200} + B_8*\text{Enroll1201-2000} + B_9*\text{Middle School} + B_{10}*\text{High School} + B_{11}*\text{PriceOfTeachers} + B_{12}*\text{PopAges5-17} + B_{13}*\text{FedShare} + B_{14}*\text{Herfindahl Index} + B_{15}*\text{Year Trend} + \text{Error}$$

Cost Model Estimation Results

Variable	Outcome: Natural log of spending per pupil
Student outcome factor score	0.178
FRPL rate	0.690
Special education rate	1.659
EL rate	0.875
Enrollment categories	
≤ 200	0.376
201–600	0.208
601–1,200	0.159
1,201–2,000	0.096
Proportion of enrollment in middle grades	0.548
Proportion of enrollment in high school grades	0.168
Comparable Wage Index for Teachers	-0.086
Year trend	0.025
Share of population ages 5–17	0.098
Federal revenue as share of total spending	-1.377
Herfindahl Index (sum of squared district shares of enrollment within the labor market)	0.233
Constant	9.141
Number of observations	1,597
R^2	0.337

Formula Representation of Cost Model Results

Mathematical Model

$$\ln(\text{Spending Per Pupil}) = B_0 + B_1*\text{Outcome} + B_2*\text{FRPL} + B_3*\text{Special Education} + B_4*\text{EL} + B_5*\text{Enroll<=200} + B_6*\text{Enroll201-600} + B_7*\text{Enroll601-1200} + B_8*\text{Enroll1201-2000} + B_9*\text{Middle School} + B_{10}*\text{High School} + B_{11}*\text{PriceOfTeachers} + B_{12}*\text{PopAges5-17} + B_{13}*\text{FedShare} + B_{14}*\text{Herfindahl Index} + B_{15}*\text{Year Trend} + \text{Error}$$

Formula Representation

$$\ln(\text{Spending Per Pupil}) = 9.14 + 0.18*\text{Outcome} + 0.69*\text{FRPL} + 1.66*\text{Special Education} + 0.88*\text{EL} + 0.38*\text{Enroll<=200} + 0.21*\text{Enroll201-600} + 0.16*\text{Enroll601-1200} + 0.10*\text{Enroll1201-2000} + 0.55*\text{Middle School} + 0.17*\text{High School} + -0.09*\text{PriceOfTeachers} + 0.10*\text{PopAges5-17} + -1.38*\text{FedShare} + 0.23*\text{Herfindahl Index} + 0.03*\text{Year Trend} + \text{Error}$$

Example Prediction for Manchester

We applied the estimated cost model formula to each district but set the outcome level for each district as well as the efficiency controls (share of population ages 5-17, federal revenue share, and Herfindahl Index) to the state averages for those variables.

Values for Predicting Manchester Per Pupil Cost

Outcome	FRPL	SpecEd	EL	ENR ≤ 200	ENR 201-600	ENR 601-1,200	ENR 1,201-2,000	ENR Share Middle	ENR Share High	Staff Price Level	POP Share 5-17	Federal Revenue Share	Herfindahl Index	Year	Constant
0%	58%	19%	15%	0	0	0	0	22%	29%	0.99	14%	6%	0.13	0	9.14

State Average Values

Calculated Predicted Per-Pupil Cost for Manchester

Predicted Cost Per Pupil = exp(9.14 + 0.18*0 + 0.69*0.58 + 1.66* 0.19 + 0.88*0.15 + 0.38*0 + 0.21*0 + 0.16*0 + 0.10*0
+0.55*0.22 + 0.17*0.29 + -0.09*0.99 + 0.10*0.14 + -1.38*0.06 + 0.23*0.13 + 0.03*0) = **$22,695**

Example Prediction for Bedford

Values for Predicting Bedford Per-Pupil Cost

Outcome	FRPL	SpecEd	EL	ENR ≤ 200	ENR 201-600	ENR 601-1,200	ENR 1,201-2,000	ENR Share Middle	ENR Share High	Staff Price Level	POP Share 5-17	Federal Revenue Share	Herfindahl Index	Year	Constant
0%	5%	13%	1%	0	0	0	0	25%	34%	0.99	14%	6%	0.13	0	9.14

Calculated Predicted Per-Pupil Cost for Bedford

Predicted Cost Per Pupil = exp(9.14 + 0.18*0 + 0.69*0.05 + 1.66* 0.13 + 0.88*0.01 + 0.38*0 + 0.21*0 + 0.16*0 + 0.10*0 + 0.55*0.25+ 0.17*0.34 + -0.09*0.99 + 0.10*0.14 + -1.38*0.06 + 0.23*0.13 + 0.03*0) = **$12,918**

Key characteristics driving the lower predicted cost per pupil in Bedford compared to Manchester are lower student needs (FRPL, Special Education, and EL).

Weight Estimation

Develop Funding Formula Adjustment Weights

Objective is to develop a simpler model that can easily be used as a formula to distribute funding and updated on an annual basis.

Uses predicted costs from cost model to estimate simpler weight estimation model using a smaller set of variables as follows:

Cost= f(Student Needs, Enrollment Size, Grade Shares)

Must first exclude from cost model spending predictions those revenues that would not be distributed by funding formula (Federal Revenues and Special Education Catastrophic Aid) or might not be distributed by formula depending on policy decisions (Transportation).

Adjust Cost Model Predictions

Take out spending that would not be included in a funding formula from cost model predictions:

- Federal Revenue (~$1,500 per student in Manchester and ~$350 per student in Bedford)
- Special Education Catastrophic Aid (~$40 per student in Manchester and ~$100 per student in Bedford)
- Transportation? (~400 per student in Manchester and ~$800 per student in Bedford)

Predicted cost excluding federal revenue and catastrophic aid: **$21,260** in Manchester and **$12,605** in Bedford

Weight Estimation Model

	Including transportation	Excluding transportation
Base per pupil cost	$5,868	$4,973
Weights		
FRPL	1.49	1.80
Special education	4.29	4.99
EL	2.20	3.01
Enrollment categories		
≤ 200	1.08	1.18
201–600	0.57	0.63
601–1,200	0.43	0.51
1,201–2,000	0.24	0.28
Enrollment in middle grades	1.42	1.66
Enrollment in high school grades	0.42	0.56

Separate models run with transportation spending included and excluded from spending per pupil, respectively. Base per-pupil cost is the intercept taken directly from the weight estimation regression model. Reported weights are calculated by dividing each estimated model coefficient by the base per-pupil cost.

Calculation of Funding Per Pupil

Simulated formula funding is calculated by applying the base and weights to each district's characteristics.

Example for Manchester:

Funding Per Pupil = $5868 * (1 + 1.49*0.58 + 4.29*0.19 + 2.20*0.15 + 1.08*0 + 0.57*0 + 0.43*0 + 0.24*0 + 1.42*0.22 + 0.42*0.29) = **$20,373**

Example for Bedford:

Funding Per Pupil = $5868 * (1 + 1.49*0.05 + 4.29*0.13 + 2.20*0.01 + 1.08*0 + 0.57*0 + 0.43*0 + 0.24*0 + 1.42*0.25 + 0.42*0.34) = **$12,664**

Predicted cost from the cost model excluding federal revenue and catastrophic aid: **$21,260** in Manchester and **$12,605** in Bedford.

R^2 for the weight estimation model is 0.982, meaning that 98.2% of the variation in predicted costs are accounted for by the factors included in the weight estimation model. The mean absolute difference between predicted costs and simulated funding per pupil is $250.

Note: formulas do not compute exactly due to rounding.

DREW ATCHISON

SENIOR RESEARCH

DATCHISON@AIR.ORG

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THANK YOU

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