

Eliminating the District Size Weight “Cliffs”

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District Size Weights

In the report *Equity and Adequacy of New Hampshire School Funding: A Cost Modeling Approach* we estimated district size weights as a series of categorical variables, where districts with enrollments falling within a certain range receive a given weight. For the model that includes transportation district size weights range from 1.08 for districts with a maximum of 200 students to 0.24 for districts with 1,201 to 2,000 students. Districts with more than 2,000 students serve as the reference group to which smaller districts are compared and therefore do not receive a size weight (an enrollment weight of 0).

We chose to model district size as a series of categorical variables due to the likely non-linear relationship between cost and district enrollment as well as the sensitivity of the relationship to functional specification. In other words, when trying to model the relationship between cost and district enrollment you can get some rather odd results (particularly at either extreme of the enrollment distribution). Bruce Baker (2018) and Jesse Levin (2018) showed how non-linear district size specifications can affect size weights in cost-function models in their reviews of Taylor et al. (2018). In the Taylor model, the non-linear enrollment specification resulted in the lowest additional funding for districts with around 1,000 students. The weight increases for districts with less than 1,000 students and for those with more than 1,000 students. In other words, there is a “U” shaped relationship. Because district size is correlated with student needs, Baker (2018) indicates that the choice of this functional form likely resulted in an overestimation of cost for large districts with relatively low needs and an underestimation of costs for moderately sized districts with relatively high needs.

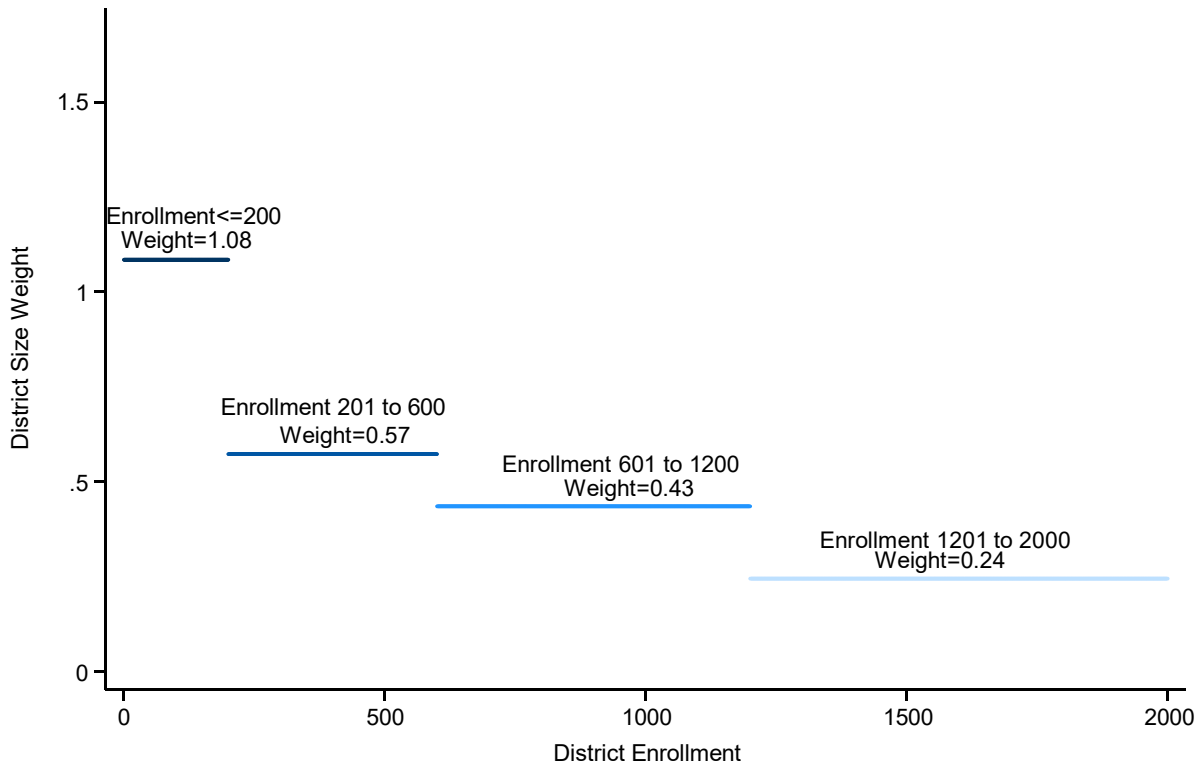
The use of categorical size categories has the advantage of not relying on a specific functional form to model the relationship between cost and enrollment and reduces the correlation between district size variables and student needs.

Although the use of categorical variables has several advantages. It also has some notable disadvantages. In particular, several members of the Commission to Study School Funding have pointed out that having categorical district size weights creates discontinuities in the amount of funding per student districts receive at the cutoffs between enrollment categories. These discontinuities, or “cliffs,” could create perverse incentives for districts to lower enrollment if they are near the enrollment cutoffs. The Commission has asked us to examine what district size weights might look like if we eliminated the cliffs between categories.

Smoothing Out the District Size Categorical Weights

Figure 1 shows the original district size categorical weights for the model that includes transportation spending. As shown, they are highest for very small districts and decrease in successive size categories. Additionally, there are clear discontinuities (cliffs) where a decrease in enrollment from one category to the next corresponds with a significant change in funding through the enrollment weights. For example, the decrease in enrollment from 201 to 200 students would change a district’s funding weight from 0.57 to 1.08, resulting in a per-pupil funding increase of \$2,993.¹

Figure 1. District Size Weights as Originally Estimated Using Model Including Transportation Spending



To eliminate the cliffs, we add a slope to each of the lines depicted in Figure 1, so that lines slope downward from left to right rather than be flat. In other words, we can pivot the lines about the average enrollment in each category so that the end of the line for one category meets the end of the line for the subsequent category. The series of lines that connect at their ends is called a spline and is shown in Figure 2. Rather than having four discrete weights, modeling the district size weights as a spline produces a different enrollment weight for each

¹ The per-pupil funding increase is calculated as the difference in weights multiplied by the base per-pupil amount:
 $Increase = (1.08 - 0.57) * \$5,868$

district with up to 2,000 students based on its exact enrollment. This approach is similar to that taken by Kansas (Baker & Duncombe, 2004).

Figure 2 also includes the formula for calculating the district size weights for districts within a given size range. For example, for a district with 162 students, you would use the formula for districts with up to 200 students. Plugging in 162 for the enrollment gives a weight of 0.89.

$$\text{Weight} = -0.00451 * 162 + 1.621 = 0.89$$

Figure 2. District Size Weights Estimated as Splines Using Model Including Transportation Spending

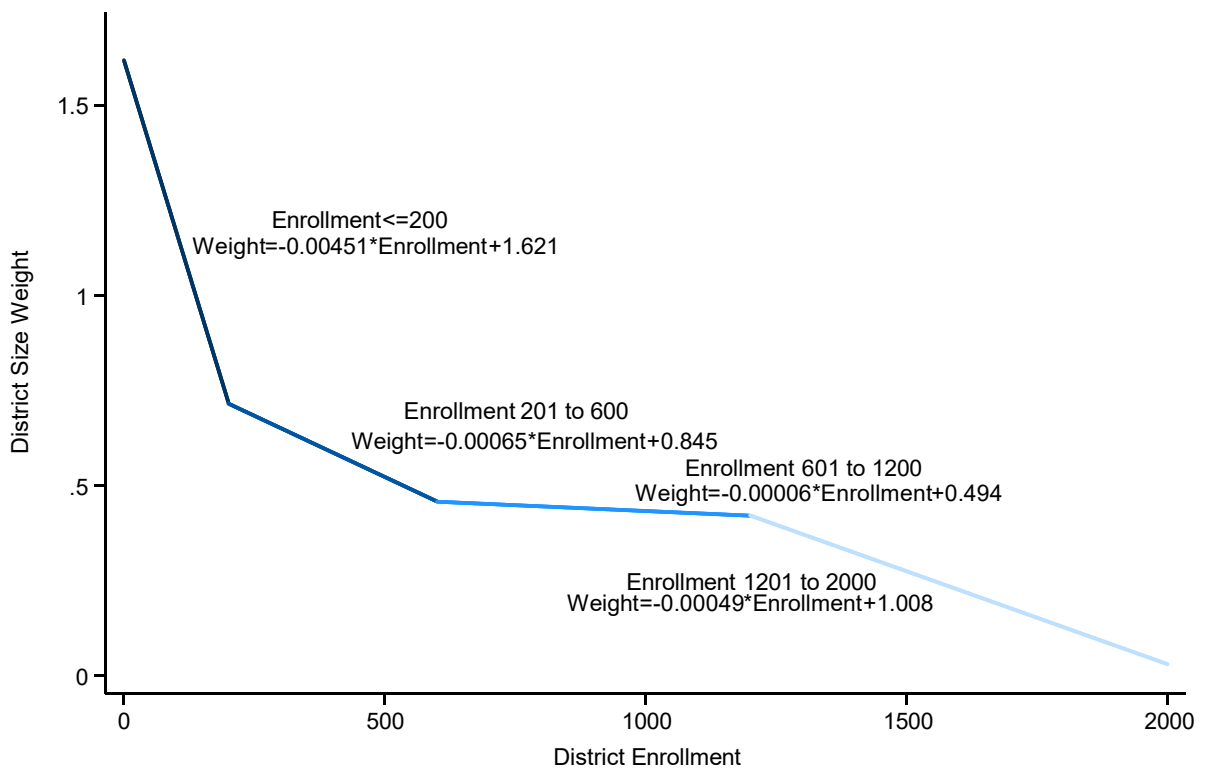


Figure 3 shows the originally estimated district size weights for the model that does not include transportation spending. Figure 4 shows the smoothed district size weights for the model that does not include transportation spending.

Figure 3. District Size Weights as Originally Estimated Using Model Not Including Transportation Spending

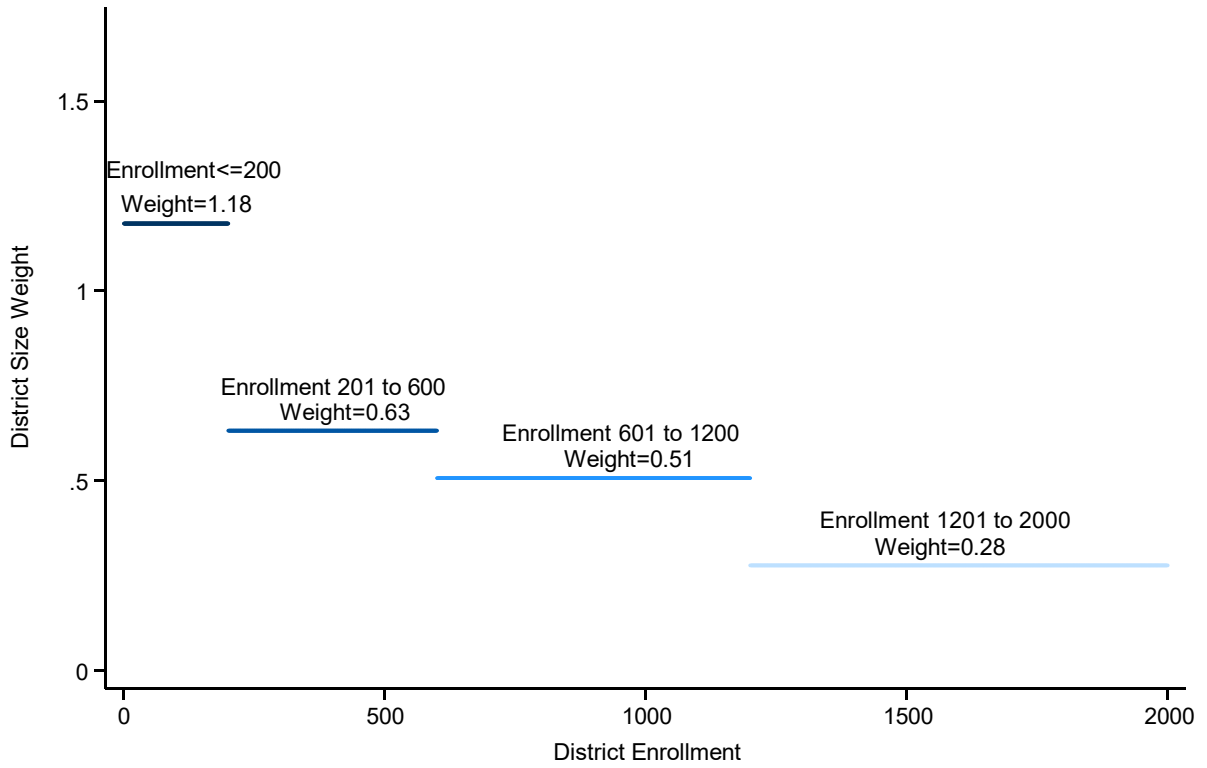
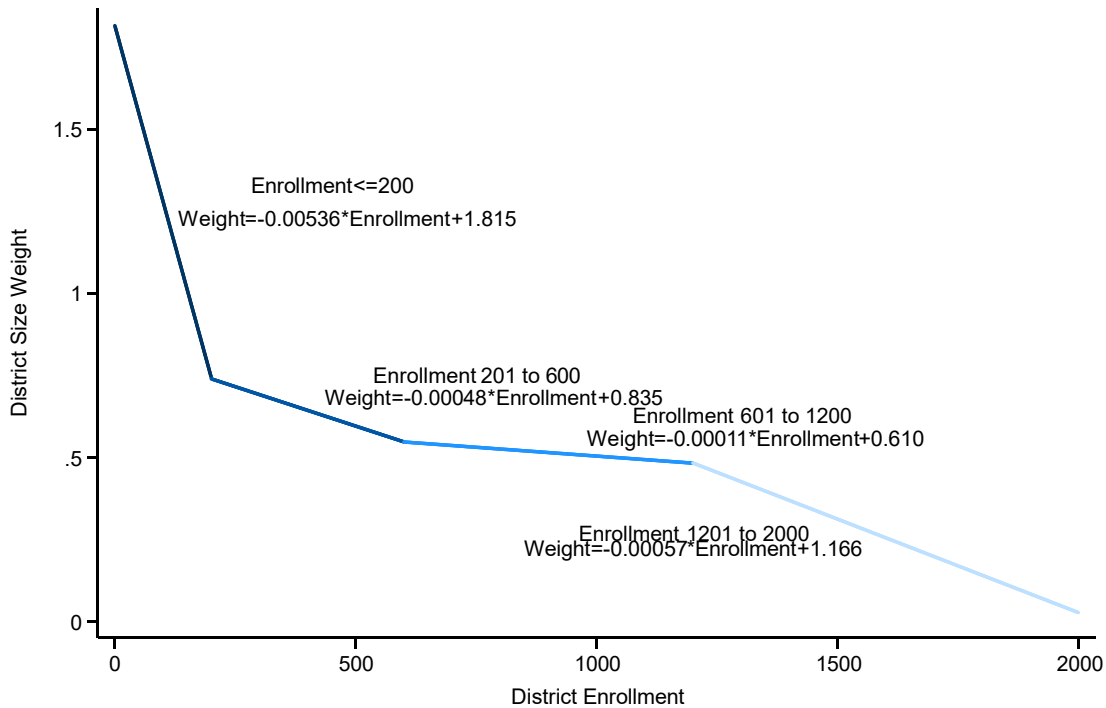


Figure 2. District Size Weights Estimated as a Spline Using Model Not Including Transportation Spending



References

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